

# **Configuration of Wireless Cooperative / Sensor Networks**

## **Final Report**

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14. ABSTRACT  This report results from a contract tasking American University in Dubai as follows: The use of spatial diversity is an effective way to counter the adverse effects of fading in wireless channels. When employing more than one antenna at each node of a wireless network is not applicable, cooperation diversity protocols exploit the inherent spatial diversity of relay channels by allowing mobile terminals to cooperate and form virtual antenna arrays. In such cooperative networks, the use of relay terminals helps to transmit information from a source node to its destination. This strategy is especially imperative in sensor networks where the transmission power for each station has to be kept to a minimum to save battery life. Depending on how a partner station is used, two classes of cooperative diversity protocols can be defined, namely static and dynamic architectures. In static protocols, M relay terminals are used, each of which forward the message they receive from the source. Here, M is a pre-determined fixed value. In dynamic configurations, on the other hand, each relay terminal forwards its received message only upon request from the destination terminal. In such cases, the number of relay terminals is not preset in advance. It is thus possible, that the source node uses one or more terminals or none at all. The purpose of this research work is to devise suitable algorithms for configuring a cooperative/sensor network.					
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# **Configuration of Wireless Cooperative / Sensor Networks**

## **Final Report**

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## **Executive Summary**

This document is submitted as the final report for EOARD grant 073040, entitled "configuration of wireless cooperative/sensor networks".

Our accomplishments during the course of this project are as follows:

- An optimal power allocation methodology for single-hop multi-branch networks is developed, so that the total transmission power for the source and all relays is minimized, subject to the symbol error rate at the destination remaining below a prescribed value. Simulation results show that this method lowers the overall transmission power considerably and results in significant extension of the network lifetime.
- A network lifetime maximization scheme is developed, such that, when the battery power of a relay is depleted, the node is removed from the set of the cooperative terminals. The signal is then routed by utilizing other nodes are relays in a new cooperative formation. Simulation results show that applying our proposed technique results in a modest increase in network lifetime.
- Three configuration algorithms for single-hop multi-branch wireless cooperative / sensor networks are developed, which incorporate optimal power allocation, network lifetime maximization, or both. These algorithms are referred to as OP, EP-LM and OP-LM, respectively.
- A method for optimal power allocation to network nodes is developed, such that the system error rate is minimized subject to total transmission power remaining constant. Simulation results for a network with 5 relay stations show a modest gain (around 1 dB) when the power allocated to each node is set optimally.
- An optimal power allocation methodology is developed for multi-branch multi-hop networks, such that the total transmission power is minimized, subject to the SER remaining below a given value. The solution to the special case of single-branch multi-hop networks is also found.
- SER minimization problem over allocated power to each relay (subject to total power remaining fixed) is extended to multi-hop networks.
- The problem of selecting the optimal cooperative route in multi-hop networks is investigated. Here, we calculate the link cost for three types of communication links, namely point-to-point, broadcast and multi-point to point links. Using these link costs, the optimal route, which has the lowest cost (or transmission power), can be found using a dynamic programming approach. Simulation results for a simple line network show that the cooperative route selection algorithm provides substantial gains in terms of the required transmission power.
- An alternate routing strategy is investigated, where at a given stage, cooperation is taken to be only from the last U stages, rather than from all the previous stages (starting from the source). This approach results in significant additional savings in terms of transmission power.

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# 1. Introduction

Space-time codes over multiple antenna systems provide diversity and coding gains over wireless fading channels. When employing more than one antenna at each node of a wireless network is not feasible, data transmission from a source node to the destination can be carried out using relay terminals. Such a cooperative strategy in effect forms a virtual antenna array with potentially substantial diversity gains.

In a cooperative network, the relay stations could simply amplify and re-transmit their received signals. The transmission power in such an amplify-and-forward (AF) approach could be fixed or dynamically adjusted depending on the message route. In contrast, the relay terminals could detect, re-modulate and re-transmit the data symbols in what is referred to as the decode-and-forward (DF) strategy. Clearly, in this case, the processing required at each node to detect data symbols would require additional battery power.

Of special importance in the development of cooperative networks is the design of efficient and practical algorithms for network configuration. This means that, for transmitting data from a source to a destination, a set of terminals need to be selected as relays, so that a network QoS measure such as symbol error rate (SER), transmission delay or system lifetime is optimized. In resource-constrained networks such as wireless sensor networks (WSN), the advantages of cooperation can be further exploited by optimally allocating the energy and bandwidth resources among users based on the available channel state information (CSI) at each node. In this research work, our goal is to investigate various configuration strategies for wireless cooperative / sensor networks.

In Section 2, we investigate single-hop multi-branch networks, in which the data is transmitted from the source to relays, which in turn retransmit the information to the destination. We will extend the work that we had previously reported in [Vaz05], [Vaz07], in which the transmission power of source and all relays were taken to be equal. A minimum number of relay terminals were selected to yield a prescribed SER. Here, we will consider two strategies to enhance the performance of this algorithm. First, we derive an expression for the optimal allocation of transmission power amongst the selected relay nodes, so that the total network power for a given system SER is minimized. Next, we model the gradual depletion of the battery power in each relay, and modify the configuration algorithm so that the relay terminals whose remaining power are below a certain minimum value are removed from the cooperative scheme. This would extend the overall lifetime of the network, since alternate routes are then selected for data transmission. We will also investigate optimal power allocation to relay stations so that the overall system error rate is minimized, subject to keeping the total transmit power constant.

In Section 3, we investigate multi-hop topologies in cooperative/sensor networks. We will extend the optimal power allocation technique to the case of multi-hop multi-branch structures, where the total transmit power is minimized subject to a required error rate at the destination. We will also study the problem of minimizing the system SER in multi-hop networks over allocated power to relay nodes, subject to total power remaining constant.

In Section 4, we study optimal routing strategies in multihop cooperative networks. In devising suitable routing techniques, we need to take various transmission mechanisms in such a setting into account. These include point-to-point links, broadcast (point-to-multipoint) links and multipoint-to-point cooperative links. As in [Kha07], we will calculate the required power (or cost) of each of these communication links, but unlike [Kha07] which uses signal-to-noise ratio as the QoS measure, we will compute the link costs so that a prescribed error rate is maintained. With individual link costs determined, the route which minimizes the total communication cost is selected as the optimal route using a dynamic programming approach. We will present simulation results to demonstrate the energy saving offered by the proposed approach over a non-cooperative scheme in which data is simply passed from one to node to the next.

## 2. Single-Hop Wireless Cooperative Networks

### 2.1 Equal Power Allocation (EP Algorithm)

In this section, we briefly review the optimal configuration algorithm for a multi-branch single-hop network in which the transmission power of the source and all the relay stations are taken to be fixed and equal [Vaz05], [Vaz07]. Figure 1 shows a generic model of a multi-branch cooperative network in which  $M$  relay terminals are to be selected out of  $N$  available sensors. The distance between every two nodes in the network is specified as  $d_{pq}$ , where  $p$  or  $q$  could represent the source,  $s$ , destination,  $d$ , or relay terminals. We also denote the channel fading coefficient for each link as  $h_{pq}$ , which are considered to be independent complex Gaussian random variables, each with zero mean and variance  $\Omega_{pq}$ , and remain unchanged during a symbol period. In addition, all noise signals are assumed to be additive white Gaussian with zero mean and variance  $N_0$ .

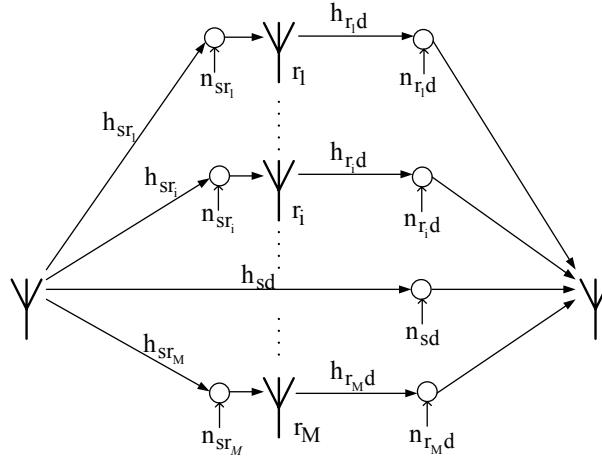


Figure 1: System model

To obtain an error rate formula, we consider an amplify-and-forward model for the relay nodes, and assume that the source and relay terminals transmit their signals through orthogonal channels. The receiver then applies maximal ratio combining (MRC) on the  $M+1$  independent copies of transmitted signal to detect the transmitted symbols. With  $M$  relay terminals, the average symbol error rate (SER) at high signal to noise ratios (SNR) is given by [Rib04]:

$$p_e(M) = \frac{c(M)}{k^{M+1}} \frac{1}{\bar{\gamma}_{sd}} \prod_{i=1}^M \left( \frac{1}{\bar{\gamma}_{sr_i}} + \frac{1}{\bar{\gamma}_{r_i d}} \right) \quad (2.1)$$

where

$$c(M) = \prod_{j=1}^{M+1} \frac{(2j-1)}{(2(M+1)!)} \quad (2.2)$$

and  $k$  is a constant which depends on the modulation type. In addition,  $\bar{\gamma}_{pq}$  is defined as  $\bar{\gamma}_{pq} = \Omega_{pq} \mathcal{E}_s / N_0$  where  $\mathcal{E}_s$  represents the average transmit power. Using the path loss model for wireless channels, for any two nodes,  $p$  and  $q$ , we can write  $\Omega_{pq} = A / d_{pq}^\nu$ , where  $A$  is a constant and  $\nu$  is the path loss exponent. Thus  $\bar{\gamma}_{pq}$  can be written as  $\bar{\gamma}_{pq} = A \mathcal{E}_s / (d_{pq}^\nu N_0)$  and hence,

$$p_e(M) = \frac{c(M)}{k^{M+1}} \frac{1}{\bar{\gamma}_{sd}^{M+1}} F_M \quad (2.3)$$

where  $F(M) = \prod_{i=1}^M m_{r_i}$  and  $m_{r_i}$  is a metric which is related to the location of the relay terminal. It is defined as:

$$m_{r_i} = \rho_i^\nu + \eta_i^\nu = (x_{n_i}^2 + y_{n_i}^2)^{\nu/2} + ((1 - x_{n_i})^2 + y_{n_i}^2)^{\nu/2} \quad (2.4)$$

where  $\rho_i = d_{sr_i} / d_{sd}$ ,  $\eta_i = d_{r_i d} / d_{sd}$ ,  $x_{n_i} = x_i / d_{sd}$  and  $y_{n_i} = y_i / d_{sd}$ .

From Equation (2.3), for a desired system SER,  $p_{e_{des}}$ ,  $M$  relays should be selected so that the following condition on the product of their metrics is satisfied:

$$m_{r_1} m_{r_2} \dots m_{r_M} \leq \frac{k^{M+1} \bar{\gamma}_{sd}^{M+1} p_{e_{des}}}{c(M)} = B_M \quad (2.5)$$

The above equation is indeed the basis for the configuration of the network. Let us suppose that the desired system error rate is denoted by  $\bar{p}_{e_{des}}$ . Initially, no relay station is considered. The resulting SER is calculated using Equation (2.3) and if the required error rate performance is not achieved, one relay terminal will be utilized. To select the most suitable relay terminal, the metric of all terminals are calculated and sorted in an ascending order. The terminal with the lowest metric from the available set is then selected. The system SER is checked again, and, if needed, additional stations will be added one at a time until the prescribed SER is achieved. The flowchart of the EP algorithm is shown in Figure 2. The reader is referred to [Vaz05] and [Vaz07] for a through description and analysis of this algorithm.

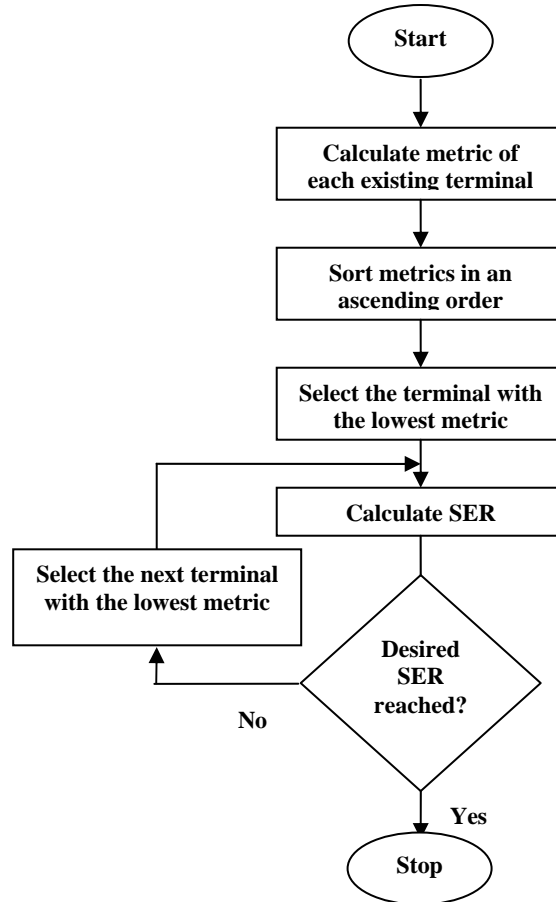


Figure 2: Flow chart of the EP algorithm

## 2.2 Optimal Power Allocation for SER-Constrained Cooperative Networks

In this section, we derive an expression for the transmission power of each relay in a cooperative formation, so that the total power is minimized, subject to the system error rate requirement at the destination being met. In [Che05] and [Hua07], the received instantaneous SNR at the destination is considered as the primary QoS. However, symbol error rate is a more meaningful metric to be used as the QoS since it directly measures the system performance.

We again consider a wireless cooperative / sensor network with one source node  $s$ , one destination node  $d$ , and  $N$  passive nodes that have a capability of serving as relays. Here, the term passive is used to imply that such nodes do not have their own information to transmit and they are only used as relays to retransmit the source node messages. Each passive node is powered by a battery with initial energy  $E_{in}$ . We consider an amplify-and-forward model for the cooperative network, where each relay station re-transmits its received signal after amplification. Here, without loss of generality, we assume unit-variance noise at the destination and relays.

Below we derive an expression for the optimal transmission power of each relay. We start out by writing the system error probability (SEP) at high signal to noise ratios when  $M$  relay terminal are utilized. This expression is given by: [Rib04]

$$\text{SER} = \frac{C(M)}{k^{M+1}} \frac{1}{P_s \Omega_{sd}} \prod_{r=1}^M \left( \frac{1}{P_s \Omega_{sr}} + \frac{1}{P_r \Omega_{rd}} \right) \quad (2.6)$$

where  $P_s$  and  $P_r$  are the transmission powers at the source node and the  $r^{\text{th}}$  relay, respectively, and  $C(M)$  is again defined as

$$C(M) = \frac{\prod_{j=1}^{M+1} (2j-1)}{2(M+1)!} \quad (2.7)$$

Given the availability of the channel state information (CSI), the optimal power allocation problem can be formulated as:

$$\min \sum_{r=1}^M P_r \quad (2.8)$$

$$\text{s.t. } \frac{C(M)}{k^{M+1}} \frac{1}{P_s \Omega_{sd}} \prod_{r=1}^M \left( \frac{1}{P_s \Omega_{sr}} + \frac{1}{P_r \Omega_{rd}} \right) \leq \text{SER} \quad (2.9)$$

**Theorem 2.1:** The solution to the constrained optimization problem given in Equations (2.8) and (2.9), (i.e.,  $P_1^*, \dots, P_M^*$ ) is unique.

*Proof:* The objective function in (2.9) is a linear function of power allocation parameters and thus is a convex function. Hence, it is enough to prove that with

$D_f := \{P_r \in (0, \infty), r = 1, \dots, M \mid f(P_1, \dots, P_M) \leq 0\}$ ,  $f : D_f \rightarrow \Re, f(P_1, \dots, P_M)$  is a convex function. Similar to [Boy04], it can be readily verified that  $f(P_1, \dots, P_M)$  is posynomial function. By verifying that its Hessian is positive semi-definite, it can be shown that the function is convex on the non-negative orthant. ■

The optimal power allocation strategy is given in the proposition below:

**Proposition 2.1:** For the set of selected relays in the network, the transmission power for the  $r^{\text{th}}$  relay is given by:

$$P_r = \frac{P_s \Omega_{sr} C(M)}{\text{SER} k^{M+1} P_s^2 \Omega_{sd} \Omega_{rd} \Omega_{sr} - \Omega_{rd} C(M) \prod_{\substack{i=1 \\ i \neq r}}^M \left( \frac{1}{P_s \Omega_{si}} + \frac{1}{P_i \Omega_{id}} \right)} \prod_{i=1}^M \left( \frac{1}{P_s \Omega_{si}} + \frac{1}{P_i \Omega_{id}} \right) \quad (2.10)$$

for  $r = 1, \dots, M$ .

*Proof:* The Lagrangian for the constrained optimization problem in Equations (2.8)-(2.9) is given by

$$L(P_1, \dots, P_M) = \sum_{r=1}^M P_r + \lambda f(P_1, \dots, P_M) \quad (2.11)$$

where

$$f(P_1, \dots, P_M) = \frac{C(M)}{k^{M+1}} \frac{1}{P_s \Omega_{sd}} \prod_{r=1}^M \left( \frac{1}{P_s \Omega_{sr}} + \frac{1}{P_r \Omega_{rd}} \right) - \text{SER} \quad (2.12)$$

For nodes  $r = 1, \dots, M$  with non-zero transmission powers, the Kuhn-Tucker conditions are given as follows:

$$\frac{\partial}{\partial P_r} L(P_1, \dots, P_M) = 1 + \lambda \frac{\partial}{\partial P_r} f(P_1, \dots, P_M) = 0, \quad (2.13)$$

where

$$\frac{\partial}{\partial P_r} f(P_1, \dots, P_M) = -\frac{C(M)}{k^{M+1}} \frac{1}{P_s \Omega_{sd}} \frac{1}{P_r^2 \Omega_{rd}} \prod_{\substack{i=1 \\ i \neq r}}^M \left( \frac{1}{P_s \Omega_{si}} + \frac{1}{P_i \Omega_{id}} \right) \quad (2.14)$$

Using Equations (2.13) and (2.14), we have

$$P_r^2 = \lambda \frac{C(M)}{k^{M+1}} \frac{1}{P_s \Omega_{sd}} \frac{1}{P_r^2 \Omega_{rd}} \prod_{\substack{i=1 \\ i \neq r}}^M \left( \frac{1}{P_s \Omega_{si}} + \frac{1}{P_i \Omega_{id}} \right), \quad (2.15)$$

for  $r = 1, \dots, M$ .

Since strong duality condition holds for convex optimization problems [Boy04], we have

$$\lambda^* f(P_1^*, \dots, P_M^*) = 0.$$

If we assume Lagrange multiplier has a positive value, then  $f(P_1^*, \dots, P_M^*) = 0$ , which yields:

$$\text{SER} = \frac{C(M)}{k^{M+1}} \frac{1}{P_s \Omega_{sd}} \prod_{r=1}^M \left( \frac{1}{P_s \Omega_{sr}} + \frac{1}{P_r \Omega_{rd}} \right) \quad (2.16)$$

From Equations (2.15) and (2.16), we can obtain the Lagrange multiplier as

$$\lambda = \frac{P_r^2}{\text{SER}} \left( \frac{\Omega_{rd}}{P_s \Omega_{sr}} + \frac{1}{P_r} \right). \quad (2.17)$$

Substituting  $\lambda$  from (2.17) to (2.15) results in Equation (2.10). Note that using Equation (2.16) in (2.10), we can readily verify that  $P_r$  values in Equation (2.10) are always positive.

### 2.2.1 Optimum Power Allocation (OP Algorithm)

In this approach, we use the same general methodology of the EP algorithm, except that the metrics are now modified to take the difference in the allocated power to each relay into account. Following an argument similar to that given for EP algorithm in Section 2.1, from Equation (2.6), for a given SER, we need to select relay stations which have the lowest metrics, where the metric for the  $i^{\text{th}}$  node in the network is now defined as

$$g_i = \left( \frac{1}{P_s \Omega_{si}} + \frac{1}{P_i \Omega_{id}} \right) \quad (2.18)$$

In the OP Algorithm, initially, the transmit power of all nodes are set to the same value as that of the source, i.e.,  $P_s$ . We calculate the metrics of all nodes in the network and sort them in an ascending order. If  $M$  relays are to be selected, we choose  $M$  terminals with the lowest metrics. The transmission power is then set using Equation (2.10).

## 2.3 Network Lifetime Maximization

In wireless sensor networks, maximization of the network lifetime is paramount. This requires that the battery power of all relay nodes be continuously monitored. Otherwise, stations which are suitably located within the network are over-utilized and are quickly depleted of their battery power. This in turn, terminates the effective lifetime of the network, if appropriate counter-measures for alternate routing of the signals are not foreseen in the configuration algorithm.

Many of the existing work on power allocation in cooperative and sensor networks focus on minimizing the transmission power to meet the QoS constraint at the destination [Hon07]. While optimal allocation of power to each relay does indeed extend the network lifetime, we expect that incorporating additional measures for monitoring the remaining energy at all relays could provide further benefit. In this section, we first present a brief overview of approaches proposed in the literature for modeling and analysis of the network lifetime. We then describe our approach for extending the lifetime of the network.

### 2.3.1 Overview of Modeling Lifetime Constraints in WSN's

Various techniques proposed in the literature for conserving energy in ad hoc and sensor networks can be generally divided into three categories, namely topology control, power-aware routing and sleep management, as described below:

**Topology control:** Topology control methods attempt to preserve desirable properties of a wireless network through reduced transmission powers. A comprehensive survey on existing topology control schemes can be found in [Sta03]. In the scheme proposed in [Vol99], a node chooses to relay through other nodes only when less power is used. Ramanathan proposes two centralized algorithms to minimize the maximal power used per node while maintaining the (bi)connectivity of the network [Ram00]. Two distributed heuristic methods are also proposed for mobile networks in [Ram00], although they may not necessarily preserve the network connectivity. Two algorithms are proposed in [Swe02, Vik03] to maintain network connectivity while keeping the transmission power to a minimum. A topology called Localized Delaunay Triangulation is shown to have a constant stretch factor with respect to the original network [Alz03]. Li et al. propose a topology control scheme which preserves the network connectivity and has bounded node degrees [Nin03]. The problem of maximizing network lifetime under topology control is studied in [Cal03].

**Power-aware routing:** Singh et al. propose five power-aware routing metrics to reduce energy consumption and extend system lifetime [Sin98]. The implementation of a minimum energy routing protocol is discussed in [Dos02a, Dos02b]. An online power-aware routing scheme is proposed in [Li01] to optimize system lifetime. Chang and Tassiulas study the problem of maximizing the lifetime of a network with known data rates [Cha00]. Chang et al. formulate the problem of choosing routes and transmission power of each node to maximize the system lifetime as a linear programming problem and discuss two centralized routing algorithms [Cha00]. In [San04], Sankar et al. formulate maximum lifetime routing as a maximum concurrent flow problem and propose a distributed algorithm.

**Sleep management:** Recent studies show that significant energy savings can be achieved by turning wireless radios off when not in use. In this approach, only a small number of nodes remain active to maintain continuous service of a network and all other nodes are scheduled to sleep. ASCENT [Cer02], SPAN [Che01], AFECA [Xu00] and GAF [Xu01] maintain network connectivity while CCP [Xia03] maintains both network connectivity and sensing coverage. More recently, a sleep schedule algorithm is proposed in [Mos05] to maximize the lifetime of network clustering.

### 2.3.2. Network Power Monitoring

In this section, we describe our proposal for extending the lifetime of a cooperative network. In most of the previous work on this subject, network lifetime is defined as the time when one or several users are depleted of energy [Che05]. However, this definition does not accurately characterize the duration in which the network operates properly in a cooperative system. A preferred way of defining the lifetime of the network is by specifying the time when the target SER at the destination cannot be achieved with a certain probability, and this is the definition that we will consider in our proposed technique.

To incorporate the residual battery life of each relay station into the configuration algorithm, we monitor the remaining energy at each node after each transmission, and if this value falls below a certain minimum, we remove the node from

the cooperative scheme. Specifically, the remaining energy at a given relay node,  $r_i$ , is denoted by  $e_{r_i}(n)$ . After the transmission of  $n^{\text{th}}$  message, the remaining energy at the selected relays is written as

$$e_{r_i}(n) = e_{r_i}(n-1) - P_s, i = 1, \dots, M. \quad (2.19)$$

We require that each relay station in a given configuration satisfy the following condition:

$$e_{r_i}(n) > P_s, i = 1, \dots, M \quad (2.20)$$

If this condition is met, the next data message will be transmitted by the same set of relay nodes, otherwise the depleted nodes are removed and a new set of relays are selected.

### 2.3.3 Equal Power Allocation with Network Lifetime Maximization (EP-LM Algorithm)

In this algorithm, we incorporate the network lifetime maximization strategy into the EP algorithm. Thus, the transmit power of the source and all relays are taken to be equal. This means that the metric of each node is defined in the same way as the EP algorithm. After each transmission, the residual energy at each node in the cooperative set is calculated and depleted nodes (if any) are removed. This would then require a new set of relay nodes to be selected from the remaining terminals in the network. The operation of the EP-LM algorithm is described below and depicted in the flowchart of Figure 3.

#### EP-LM Algorithm

Initialization:

$R = 1, n = 1,$   
 $P_i = P_s, \text{ for } i = 1, \dots, N$

Recursion 1:

Calculate  $h_i, i = 1, \dots, N$   
 Select  $R$  terminals that have lowest value of  $h_i$   
 Sort all  $h_r$  such that  $h_1 < h_2 < \dots < h_R$   
 if the calculated error from (3.14) is less or equal to the required SER  
     if  $P_s < E_r(n)$  for all  $r = 1, \dots, R$

Recursion 2

$n = n + 1$   
 $E_r(n) = E_r(n-1) - P_s, \text{ for } r = 1, \dots, R$   
 if  $P_s > E_r(n)$  for some  $r = 1, \dots, R$   
     Remove depleted nodes  
      $N = N - \text{number of depleted nodes}$   
     break

$R = R + 1$   
 if  $R > N$

break

### 2.3.4 Optimal Power Allocation with Network Lifetime Maximization (OP-LM Algorithm)

We now incorporate both Optimal Power allocation to the relays as well as the network Lifetime Maximization in the configuration method, which we refer to as the OP-LM algorithm. The transmission power for all the nodes is initialized to that of the source,  $P_s$ . The number of relays is also set to  $M$ . The metric for all the nodes are then calculated using the modified definition of Equation (2.18), and  $M$  terminals with the lowest metrics are selected. The optimal value of the transmission power is then calculated. After the transmission of each symbol, the residual energy of all relays are examined, and the relays with depleted battery power are replaced with terminals with sufficient energy. This procedure continues until the number of nodes with sufficient energy which yield the required SER is less than the number of remaining nodes in the network. The operation of the OP-LM algorithm is shown below.

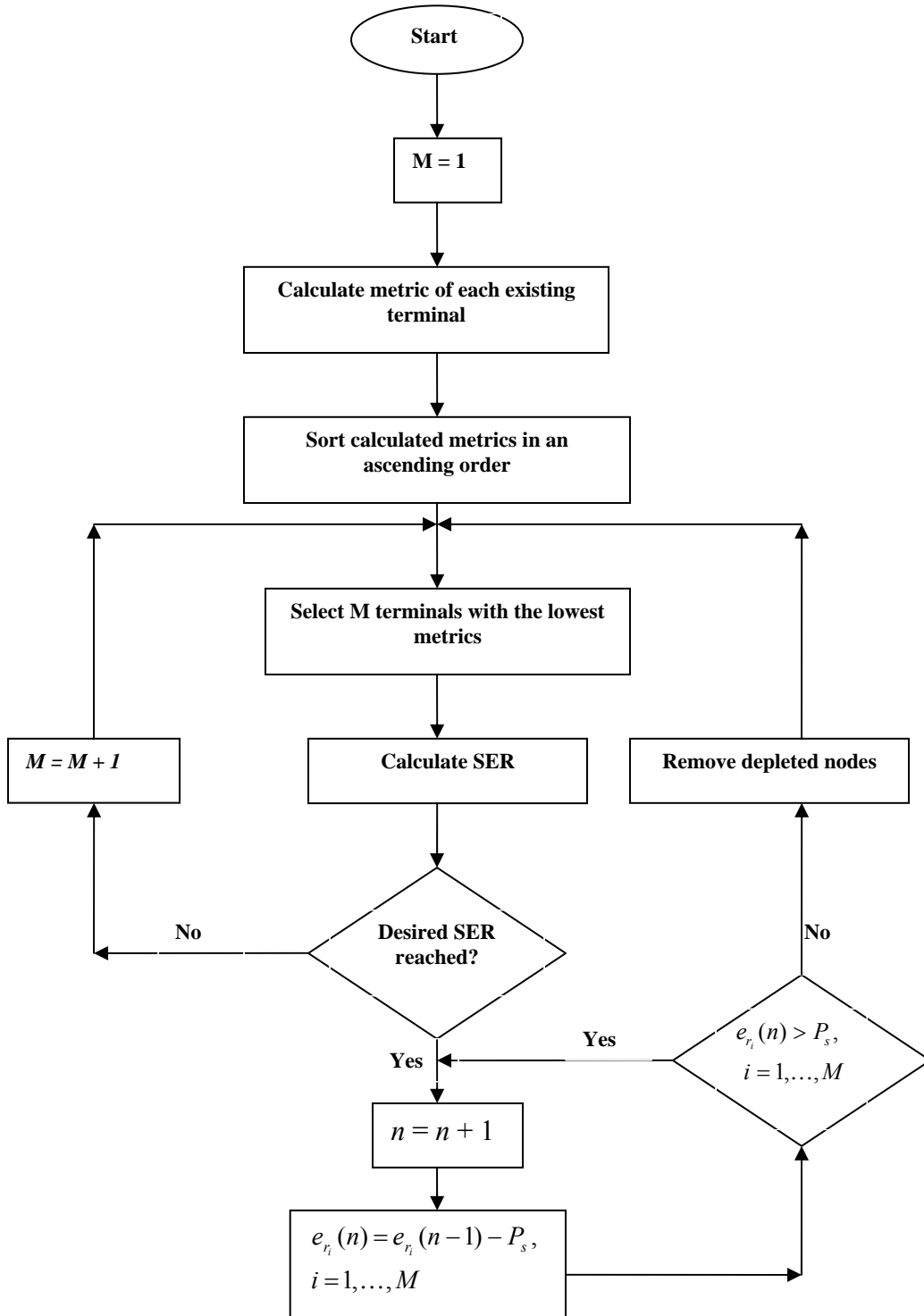


Figure 3: Flowchart of EP-LM Algorithm

## OP-LM Algorithm

### Algorithm 1:

Maximal Residual Energy strategy

Initialization:

$R = N, n = 1,$   
 $P_i = P_s, \text{ for } i = 1, \dots, N$

Recursion 1:

Calculate  $g_{lr}, r = 1, \dots, R$   
 Calculate the optimum values of  $P_r$   
 if  $P_r < E_r(n)$  for all  $r = 1, \dots, R$

Recursion 2

$n = n + 1$   
 $E_r(n) = E_r(n-1) \hat{a}^{\nu} P_r, \text{ for } r = 1, \dots, R$   
 if  $P_r > E_r(n)$  for some  $r = 1, \dots, R$   
 Remove depleted nodes  
 $R = R - \hat{a}^{\nu}$  number of depleted nodes  
 break

if  $R \hat{a}^{\nu} \approx 0$   
 break

## 2.4. Simulation Results

In this section, we present simulation results to assess the performance of configuration algorithms presented in the previous sections. The data bits are modulated using BPSK. All communication links in the network are considered to exhibit Rayleigh flat fading. The source-to-destination link is assumed to have a normalized distance of 1. We assume, (without any loss of generality) that the noise at relays and the destination node has a mean of zero and variance of 1. The QoS requirement for SER at the destination is assumed to be  $10^{-5}$ .

In Figure 4, the average transmission power from all the relays are compared for the EP and OP algorithms. For the OP algorithm, the number of the nodes that can be selected as relays is taken to be  $N = 14$ . In addition, we assume that the coefficients  $\Omega_{pq}$  fall randomly within a range of 1 to 2 with equal probability. For the path loss exponent  $\nu = 2$ , this is equivalent to  $1 < d_{pq}^2 < 1/2$ . It can be seen from the plot that the optimum power allocation scheme significantly preserves the power consumption in the network given the required SER at the destination. For example, with source power  $P_s = 20\text{dB}$ , the EP algorithm requires one relay node so that the SER at the destination is  $10^{-5}$ . This terminal is selected optimally and has the same transmission power as the source (i.e., 20 dB). The OP algorithm, on the other hand, uses 14 relays and sets the power allocated to each one optimally. By doing this, the total power used for signal transmission is substantially reduced to about 9.5 dB. Similarly, for  $P_s = 14\text{ dB}$ , the EP algorithm results in the selection of two relays, which together require a power of about 17 dB. By contrast, the OP algorithm requires a total of 10 dB for all the 14 relays to achieve the same SER at the destination. Furthermore, at very low source powers, (e.g.,  $P_s = 2\text{ dB}$ ), using the EP algorithm, we are unable to reach the required error rate performance no matter how many relays are used. The OP algorithm does indeed reach the required performance by optimal setting of the power for each node.

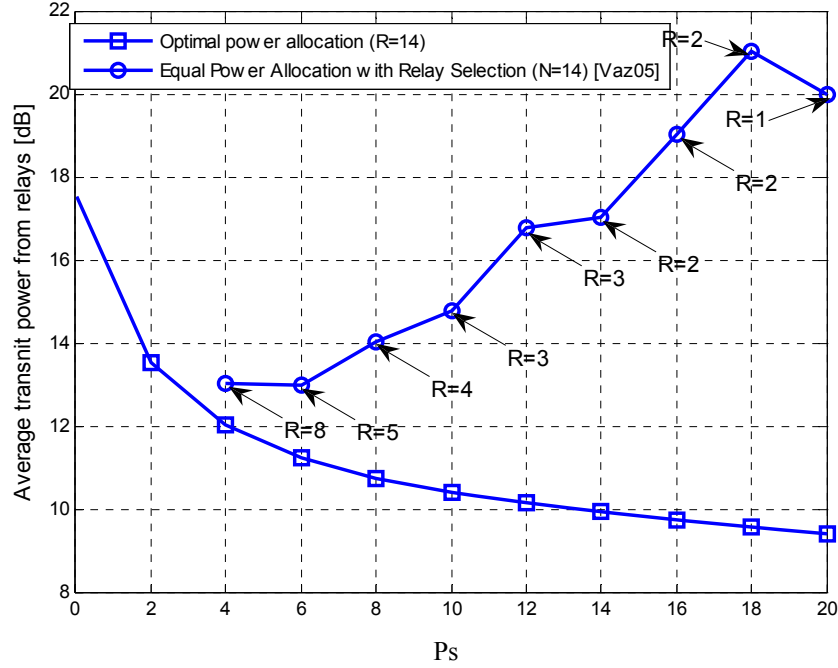


Figure 4: Performance comparison of EP and OP algorithms, in a network with  $N = 14$  potential relays and SER of  $10^{-5}$ .

In Figure 5, we compare the average network lifetime for the OP and OP-LM algorithms. Here, the average network lifetime with respect to the initial energy at each node is depicted versus the number of potential relays in the network. The initial battery energy of relays is assumed to be equal, i.e.  $E_r(0) = E_0$  for all relay stations. Specifically, we take  $E_0$  to be an integer multiple of  $P_s$ , i.e.  $E_0 = 100 P_s$ . As expected, the network lifetime in both cases increases as the number of relay nodes or the value of  $P_s$  increases. However, the plots show that the power-aware OP-LM algorithm only provides a marginal improvement over the OP technique. The simulation results reveal that the optimal power allocation plays a significant factor in reducing the total transmission power in the network and thus) extending the network lifetime. The strategy of monitoring the energy of the relay stations using the technique applied in the previous section at the relays provides only a marginal enhancement. We will explore other lifetime maximization schemes in the future.

Finally, in Figure 6, the network lifetime of the EP and EP-LM algorithms are compared for different number of relays and finite battery power at all relay nodes. Notice that as expected, the network lifetime for the EP algorithm does not change as the number relays increases since it is, in principle, a function of the transmission power of the relays. For the EP-LM algorithm, on the other hand, network lifetime does increase as the number of relays increases. However, due to the non-optimal selection of transmission power at each relay, the rate of increase in the network lifetime is modest compared to the OP-LM case. Generally, the plot shows that network lifetime is extended by applying the technique of the previous section, especially as the number of relays increases.

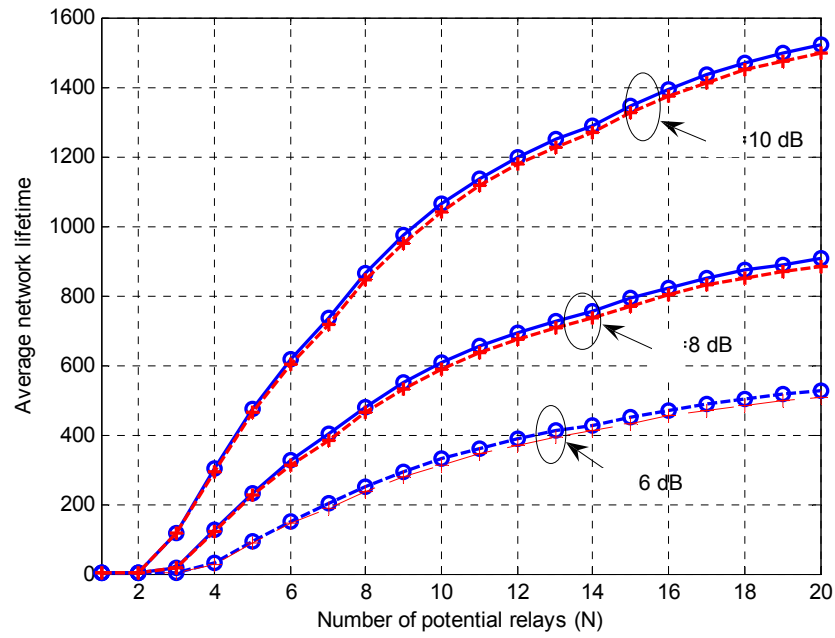


Figure 5. The lifetime performance of OP (dashed red lines) and OP-LM (solid blue lines) algorithms.

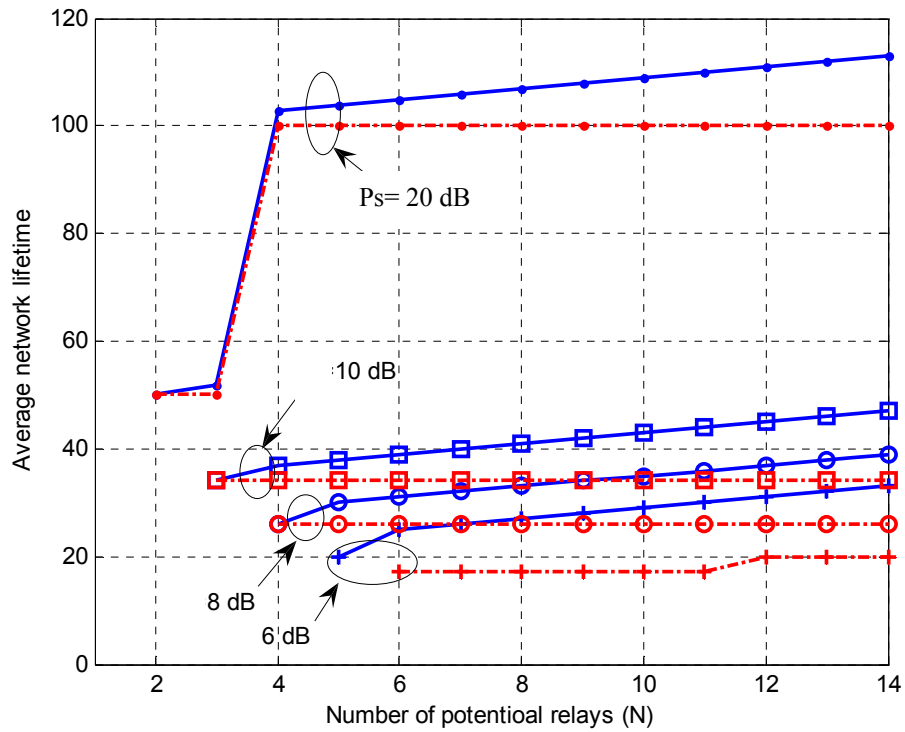


Figure 6. The lifetime performance of EP (dashed red lines) and EP-LM (solid blue lines) algorithms.

## 2.5. SER Minimization in Power-Constrained Networks

In this section, we investigate an alternate problem formulation in which the system symbol error rate for a multi-branch wireless sensor network is minimized over the power allocated to all nodes, subject to a constraint on the total transmission power.

Let us assume again, without any loss of generality, that the variance of noise at the destination and relays is one. With  $M$  relay terminals operating under the amplify-and-forward protocol, the optimization problem described above can be stated as follows:

$$\min \frac{C(M)}{k^{M+1}} \frac{1}{P_s \Omega_{sd}} \prod_{r=1}^M \left( \frac{1}{P_s \Omega_{sr}} + \frac{1}{P_r \Omega_{rd}} \right) \quad (2.21)$$

$$\text{s.t. } \sum_{r=1}^M P_r \leq P, P_r \geq 0 \quad (2.22)$$

where  $P_r, r=1, \dots, M$  correspond to the transmission power at the relay nodes and  $P$  corresponds to the total power. The error rate formula given in Equation (2.21) is valid at high signal to noise ratios [Rib04].

The optimal power allocation strategy is given in the proposition below:

**Theorem 2.2:** For the set of selected relays in the network, the transmission power for each node is given by

$$P_r = \frac{P}{\sum_{k=1}^M \sqrt{\frac{1}{\Omega_{kd}} \prod_{i=1, i \neq k}^M \left( \frac{1}{P_s \Omega_{si}} + \frac{1}{P_i \Omega_{id}} \right)}} \sqrt{\frac{1}{\Omega_{rd}} \prod_{i=1, i \neq r}^M \left( \frac{1}{P_s \Omega_{si}} + \frac{1}{P_i \Omega_{id}} \right)} \quad \text{for } r = 1, \dots, M \quad (2.23)$$

*Proof:* The Lagrangian in Equations (2.21) and (2.22) is

$$L(P_1, \dots, P_M) = h(P_1, \dots, P_M) + \lambda \left( \sum_{r=1}^M P_r - P \right) \quad (2.24)$$

where

$$h(P_1, \dots, P_M) = \frac{C(M)}{k^{M+1}} \frac{1}{P_s \Omega_{sd}} \prod_{r=1}^M \left( \frac{1}{P_s \Omega_{sr}} + \frac{1}{P_r \Omega_{rd}} \right) \quad (2.25)$$

For nodes  $r = 1, \dots, M$  with nonzero transmit powers, the Kuhn-Tucker conditions are

$$\frac{\partial}{\partial P_r} L(P_1, \dots, P_M) = \frac{\partial}{\partial P_r} h(P_1, \dots, P_M) + \lambda = 0, \quad (2.26)$$

where

$$\frac{\partial}{\partial P_r} h(P_1, \dots, P_M) = -\frac{C(M)}{k^{M+1}} \frac{1}{P_s \Omega_{sd}} \frac{1}{P_r^2 \Omega_{rd}} \prod_{i=1, i \neq r}^M \left( \frac{1}{P_s \Omega_{si}} + \frac{1}{P_i \Omega_{id}} \right) \quad (2.27)$$

Using Equations (2.26) and (2.27), we have

$$P_r^2 = \frac{1}{\lambda} \frac{C(M)}{k^{M+1}} \frac{1}{P_s \Omega_{sd}} \frac{1}{\Omega_{rd}} \prod_{i=1, i \neq r}^M \left( \frac{1}{P_s \Omega_{si}} + \frac{1}{P_i \Omega_{id}} \right), \text{ for } r = 1, \dots, M. \quad (2.28)$$

Since strong duality condition holds for convex optimization problems [Boy04], we can write

$$\lambda^* f(P_1^*, \dots, P_M^*) = 0, \quad (2.29)$$

where  $f(P_1^*, \dots, P_M^*) = \sum_{r=1}^M P_r^* - P$ . Assuming a positive value for the Lagrange multiplier, we get

$f(P_1^*, \dots, P_M^*) = 0$ , which is equivalent to  $\sum_{r=1}^M P_r^* = P$ . Thus, using Equation (2.28), we obtain the following relation:

$$\sum_{r=1}^M \sqrt{\frac{1}{\lambda} \frac{C(M)}{k^{M+1}} \frac{1}{P_s \Omega_{sd}} \frac{1}{\Omega_{rd}} \prod_{\substack{i=1 \\ i \neq r}}^M \left( \frac{1}{P_s \Omega_{si}} + \frac{1}{P_i \Omega_{id}} \right)} = P \quad (2.30)$$

from which  $\lambda$  can be calculated as follows:

$$\lambda = \left[ \sum_{r=1}^M \sqrt{\frac{1}{P^2} \frac{C(M)}{k^{M+1}} \frac{1}{P_s \Omega_{sd}} \frac{1}{\Omega_{rd}} \prod_{\substack{i=1 \\ i \neq r}}^M \left( \frac{1}{P_s \Omega_{si}} + \frac{1}{P_i \Omega_{id}} \right)} \right]^2 \quad (2.31)$$

Finally, Using  $\lambda$  from Equation (2.31) in Equation (2.28), we obtain the allocated power to each relay as given below:

$$P_r = \frac{1}{\frac{1}{P} \sum_{r=1}^M \sqrt{\frac{C(M)}{k^{M+1}} \frac{1}{P_s \Omega_{sd}} \frac{1}{\Omega_{rd}} \prod_{\substack{i=1 \\ i \neq r}}^M \left( \frac{1}{P_s \Omega_{si}} + \frac{1}{P_i \Omega_{id}} \right)}} \sqrt{\frac{C(M)}{k^{M+1}} \frac{1}{P_s \Omega_{sd}} \frac{1}{\Omega_{rd}} \prod_{\substack{i=1 \\ i \neq r}}^M \left( \frac{1}{P_s \Omega_{si}} + \frac{1}{P_i \Omega_{id}} \right)} \quad (2.32)$$

Simplification of Equation (2.32) leads to Equation (2.23).

■

It is clear that  $P_r$  values in Equation (2.23) are always positive. In addition, note that when  $\Omega_{sr} = \Omega_{rd}$ , the allocated power to all relay nodes will be the same, i.e.,

$$P_r = \frac{P}{2M}, \text{ for } r = 1, \dots, M \quad (2.33)$$

## 2.5.1 Simulations Results

To demonstrate the effectiveness of the technique presented in the previous section, we consider a network consisting of 5 relays with the following node configuration:

$$\Omega_{sr} = \frac{1}{r^3}, \quad \Omega_{rd} = 1. \quad (2.34)$$

BPSK is used for the modulation of the data symbols. Notice that Equation (2.23) provides a set of relations which need to be satisfied by the optimal power coefficients. To arrive at the final values of the transmit power for the nodes, we use an iterative procedure. For the initial values, we divide the available power equally between the relays.

In Figure 7, the BER for the multibranch system is plotted vs. SNR for the case in which the transmit powers are set optimally according to Equation (2.23). The plot of BER vs. SNR for the case of equal-power allocation for all nodes is also shown for comparison. Notice that at high SNR's (for which the error rate equation is valid), the optimal approach yields a gain of about 1 dB. We expect similar gains to be achieved for other network configurations.

As mentioned above, the final values of the allocated power to all nodes are obtained iteratively. Figure 8(a) demonstrates the convergence of power allocated to the first relay node. Notice that after about 8 iterations, the allocated power to the nodes becomes stable. Figure 8(b) the steady-state value of the allocated power to all nodes.

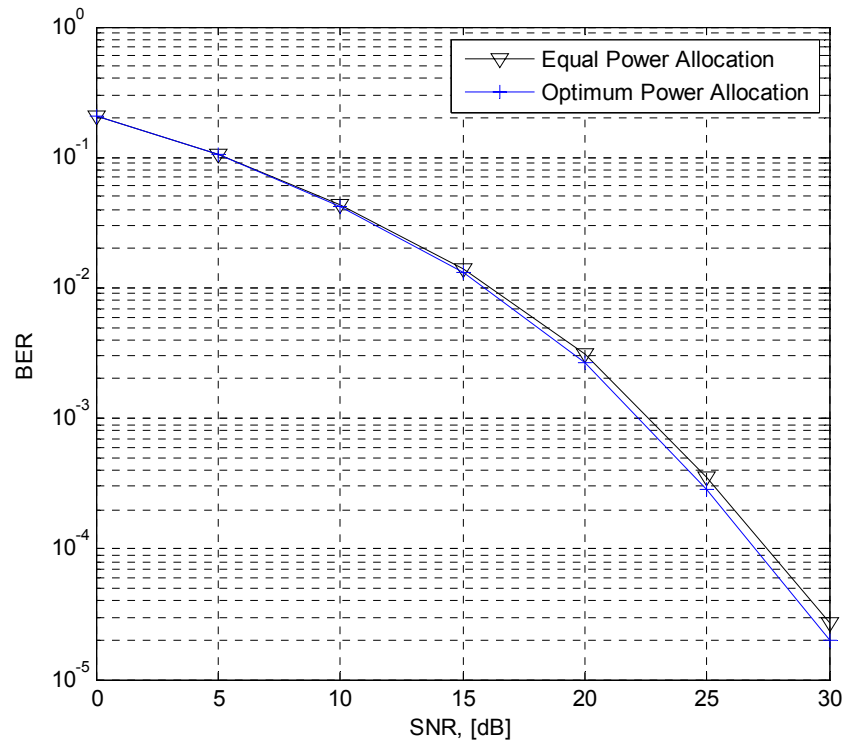


Figure 7: BER vs. SNR in power-constrained networked

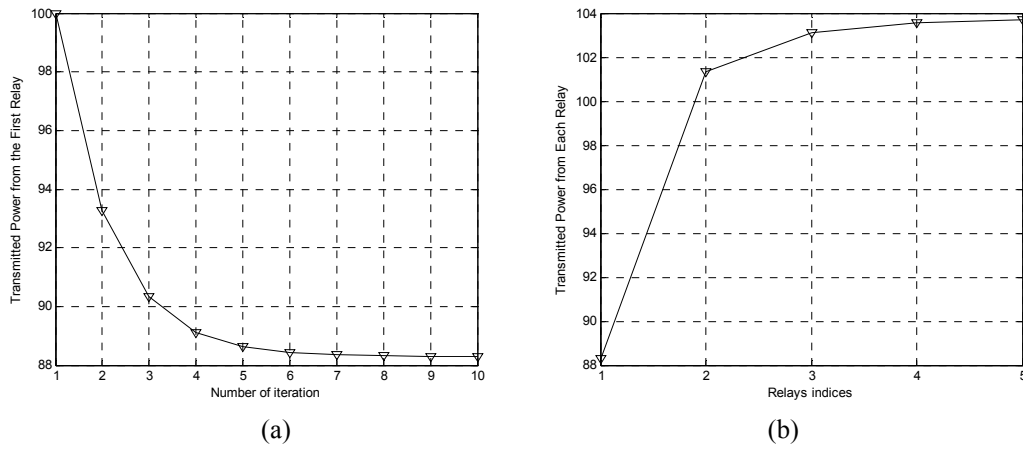


Figure 8: (a) Convergence of the allocated power to the first relay; (b) Steady-state value of the power allocated to each relay

### 3. Multihop Wireless Cooperative Networks

In this section, we extend the power optimization technique derived in Section 2.2 for multi-branch single-hop networks to multi-hop cooperative structures. We will then obtain the solution for the (special case of) single-branch multihop networks. In Section 3.3, we investigate the problem of SER minimization over the power allocated to individual nodes when the total transmission power is fixed.

#### 3.1. Power Optimization in Multi-Branch, Multihop Relay Networks

In this section, we derive the optimal power allocation to all nodes in a multi-branch multi-hop network, given a required error rate at the destination.

Let us consider that the network consists of  $M$  branches from the source to the destination, where each branch is composed of  $M_r$  hops,  $r = 1, \dots, M$ . Without any loss of generality, we assume unit-variance noise at the destination and relays. With relay terminals operating under the amplify-and-forward protocol, the system error probability at high signal to noise ratios is given by [Rib04]

$$\text{SER} = \frac{C(M)}{k^{M+1}} \frac{1}{P_0 \Omega_0} \prod_{r=1}^M \left( \frac{1}{P_0 \Omega_{r,0}} + \sum_{m=1}^{M_r} \frac{1}{P_{r,m} \Omega_{r,m}} \right) \quad (3.1)$$

where  $P_0$  and  $P_{r,m}$  are the transmitted power at the source node and the relay in the  $r^{\text{th}}$  branch and  $m^{\text{th}}$  hop, respectively. In addition,  $C(M)$  is defined as

$$C(M) = \frac{\prod_{j=1}^{M+1} (2j-1)}{2(M+1)!} \quad (3.2)$$

In addition,  $k$  is a constant which depends on the type of modulation used (For M-PSK,  $k = 2 \sin^2(\pi/M)$ ). For any two nodes,  $p$  and  $q$ ,  $\Omega_{r,m} = 1/d_{p,q}^v$ , where  $d_{p,q}$  is the distance between nodes  $p$  and  $q$ , and  $v$  is the path-loss exponent.  $\Omega_0$  is the term corresponding to the source-destination link.

Unlike [Rib04] and [Vaz05], in which uniform power allocation is considered among the source and relays, here we minimize the total transmit power over the power allocated to each node, subject to the SER at the destination being below a prescribed value. Given the knowledge of channel coefficients, the optimal power allocation problem can be formulated as

$$\min \sum_{r=1}^M \sum_{m=1}^{M_r} P_{r,m} \quad (3.3)$$

$$\text{s.t. } \frac{C(M)}{k^{M+1}} \frac{1}{P_0 \Omega_0} \prod_{r=1}^M \left( \frac{1}{P_0 \Omega_{r,0}} + \sum_{m=1}^{M_r} \frac{1}{P_{r,m} \Omega_{r,m}} \right) \leq \text{SER} \quad (3.4)$$

**Theorem 3.1:** The solution to the constrained optimization problem given above, i.e.,  $P_1^*, \dots, P_M^*$  is unique.

**Proof:** The objective function in Equation (3.3) is a linear function of power allocation parameters and thus is a convex function. Hence, it is enough to prove that with

$D_f := \{P_{r,m} \in (0, \infty), r = 1, \dots, M \mid f(\{P_{r,m}\}_{(r,m) \in \Omega}) \leq 0\}$ ,  $f : D_f \rightarrow \mathbb{R}$ ,  $f(\{P_{r,m}\}_{(r,m) \in \Omega})$  is a convex function. Similar to [Boy04], it can be readily verified that  $f(\{P_{r,m}\}_{(r,m) \in \Omega})$  is *posynomial* function. However, since its Hessian can be shown to be positive semi-definite, the function will be convex in on the nonnegative orthant.

■

The optimal power allocation strategy is shown in the proposition below:

**Proposition 3.1:** For the set of selected relays in the network, the optimal transmission power for the node in the  $r^{\text{th}}$  branch and  $m^{\text{th}}$  hop is given by

$$P_{r,m} = \frac{C(M)}{\text{SER } k^{M+1} P_0 \Omega_0 \Omega_{r,m} - C(M) \left( \frac{\Omega_{r,m}}{P_0 \Omega_{r,0}} + \sum_{\substack{j=1 \\ j \neq r}}^{M_r} \frac{\Omega_{r,m}}{P_{r,j} \Omega_{r,j}} \right) \prod_{\substack{i=1 \\ i \neq r}}^M \left( \frac{1}{P_0 \Omega_{i,0}} + \sum_{j=1}^{M_i} \frac{1}{P_{i,j} \Omega_{i,j}} \right)} \prod_{\substack{i=1 \\ i \neq r}}^M \left( \frac{1}{P_0 \Omega_{i,0}} + \sum_{j=1}^{M_i} \frac{1}{P_{i,j} \Omega_{i,j}} \right)$$

$$r = 1, \dots, M, \quad m = 1, \dots, M_r. \quad (3.5)$$

**Proof:** The Lagrangian for the constrained optimization problem in Equations (3.3) and (3.4) is given by

$$L(\{P_{r,m}\}_{(r,m) \in \Omega}) = \sum_{r=1}^M \sum_{m=1}^{M_r} P_{r,m} + \lambda f(\{P_{r,m}\}_{(r,m) \in \Omega}) \quad (3.6)$$

where the set  $\Omega$  is defined as

$$\Omega = \{(i, j) \mid i = 1, 2, \dots, M, j = 1, 2, \dots, M_i\}, \quad (3.7a)$$

and

$$f(\{P_{r,m}\}_{(r,m) \in \Omega}) = \frac{C(M)}{k^{M+1}} \frac{1}{P_0 \Omega_0} \prod_{r=1}^M \left( \frac{1}{P_0 \Omega_{r,0}} + \sum_{m=1}^{M_r} \frac{1}{P_{r,m} \Omega_{r,m}} \right) - \text{SER} \quad (3.7b)$$

For nodes  $r = 1, \dots, M, m = 1, \dots, M_r$  with nonzero transmit powers, the Kuhn-Tucker conditions are given by

$$\frac{\partial}{\partial P_{r,m}} L(\{P_{r,m}\}_{(r,m) \in \Omega}) = 1 + \lambda \frac{\partial}{\partial P_{r,m}} f(\{P_{r,m}\}_{(r,m) \in \Omega}) = 0, \quad (3.8)$$

where

$$\frac{\partial}{\partial P_{r,m}} f(\{P_{r,m}\}_{(r,m) \in \Omega}) = -\frac{C(M)}{k^{M+1}} \frac{1}{P_0 \Omega_0} \frac{1}{P_{r,m}^2 \Omega_{r,m}} \prod_{\substack{i=1 \\ i \neq r}}^M \left( \frac{1}{P_0 \Omega_{i,0}} + \sum_{j=1}^{M_i} \frac{1}{P_{i,j} \Omega_{i,j}} \right) \quad (3.9)$$

Using Equations (3.8) and (3.9), we have

$$P_{r,m}^2 = \lambda \frac{C(M)}{k^{M+1}} \frac{1}{P_0 \Omega_0} \frac{1}{\Omega_{r,m}} \prod_{\substack{i=1 \\ i \neq r}}^M \left( \frac{1}{P_0 \Omega_{i,0}} + \sum_{j=1}^{M_i} \frac{1}{P_{i,j} \Omega_{i,j}} \right), \quad (3.10)$$

$$r = 1, \dots, M, \quad m = 1, \dots, M_r.$$

Since strong duality condition holds for convex optimization problems [Boy04], we have  $\lambda^* f(\{P_{r,m}^*\}_{(r,m) \in \Omega}) = 0$ . If we assume Lagrange multiplier has a positive value, we obtain the following relation:  $f(\{P_{r,m}^*\}_{(r,m) \in \Omega}) = 0$ , which is equivalent to

$$\text{SER} = \frac{C(M)}{k^{M+1}} \frac{1}{P_0 \Omega_0} \prod_{r=1}^M \left( \frac{1}{P_0 \Omega_{r,0}} + \sum_{m=1}^{M_r} \frac{1}{P_{r,m} \Omega_{r,m}} \right) \quad (3.11)$$

Using Equations (3.10) and (3.11), we can find the Lagrange multiplier as

$$\lambda = \frac{P_{r,m}^2}{\text{SER}} \left( \frac{\Omega_{r,m}}{P_0 \Omega_{i,0}} + \sum_{j=1}^{M_i} \frac{\Omega_{r,m}}{P_{r,j} \Omega_{r,j}} \right). \quad (3.12)$$

Substituting  $\lambda$  from (3.12) to (3.10) results in  $P_{r,m}$  given below:

$$P_{r,m} = \frac{C(M)}{\text{SER} k^{M+1} P_0 \Omega_0 \Omega_{r,m} - C(M) \left( \frac{\Omega_{r,m}}{P_0 \Omega_{r,0}} + \sum_{\substack{j=1 \\ j \neq r}}^{M_r} \frac{\Omega_{r,m}}{P_{r,j} \Omega_{r,j}} \right) \prod_{\substack{i=1 \\ i \neq r}}^M \left( \frac{1}{P_0 \Omega_{i,0}} + \sum_{j=1}^{M_i} \frac{1}{P_{i,j} \Omega_{i,j}} \right)} \prod_{\substack{i=1 \\ i \neq r}}^M \left( \frac{1}{P_0 \Omega_{i,0}} + \sum_{j=1}^{M_i} \frac{1}{P_{i,j} \Omega_{i,j}} \right)$$

It can be shown that the  $P_{r,m}$  calculated using the above expression is always positive. ■

### 3.2. Power Optimization in SER-Constrained Multihop Links

In the previous section, we derived the optimal power allocation for the general case of a multi-branch multihop network. In this section, we consider the (special) case of a single-branch multihop network.

We consider that the network consists of a source node and  $M$  cooperative nodes that relay the source message in  $M$  successive time slots. Without any loss of generality, we assume unit-variance noise at the destination and relays. The constrained optimization problem can thus be formulated as follows:

$$\min \sum_{m=0}^M P_m, \quad (3.13)$$

$$\text{s.t. } \frac{C(1)}{k^2} \sum_{m=0}^M \frac{1}{P_m \Omega_m} \leq \text{SER}, \quad (3.14)$$

where  $P_0$  and  $P_m$ ,  $m=1, \dots, M$  are the allocated transmission power to the source and the relays, respectively. In addition,  $C(1)=3/4$  and  $k$  is a constant whose value depends on the type of modulation used. (For BPSK modulation,  $k=2$ .)

Setting the derivative of Lagrangian in Equations (3.13) and (3.14) to zero, we get

$$1 - \lambda \frac{3}{4k^2} \frac{1}{\Omega_m P_m^2} = 0. \quad (3.15)$$

which yields

$$\lambda = \frac{4k^2}{3} \Omega_m P_m^2. \quad (3.16)$$

Using Equations (3.14) and (3.16), we have

$$\sum_{m=0}^M \frac{2k\Omega_m^{1/2}}{\sqrt{3\lambda\Omega_m}} = \frac{4k^2}{3} \text{SER}, \quad (3.17)$$

and therefore,

$$\lambda = \frac{3}{4k^2 \text{SER}^2} \left( \sum_{i=0}^M \frac{1}{\Omega_i^{1/2}} \right)^2. \quad (3.18)$$

Using  $\lambda$  from Equation (3.18) in Equation (3.16), we can find the allocated power for the  $M^{\text{th}}$  relay in  $M$ -hop wireless system as given below:

$$P_m = \frac{3}{4k^2 \text{SER} \Omega_m^{1/2}} \left( \sum_{i=0}^M \frac{1}{\Omega_i^{1/2}} \right)^2. \quad (3.19)$$

### 3.3. SER Minimization in Power-Constrained Multihop Links

In the previous section, total transmission power was minimized over the allocated power to individual nodes under the constraint that the system SER remains below a given value. Below, we state an alternate problem formulation for a multihop relay network, in which the overall system error rate is to be minimized over allocated power to nodes, subject to the total transmission power remaining below a prescribed value. This constrained optimization problem can be stated as follows:

$$\min \sum_{m=0}^M \frac{1}{\Omega_m P_m} \quad (3.20)$$

$$\text{s.t. } \sum_{m=0}^M P_m \leq P \quad (3.21)$$

We will once again use the Lagrange multipliers technique to solve the above constrained optimization problem. Setting the derivative of Lagrangian in Equations (3.20) and (3.21) to zero, we have

$$\frac{-1}{\Omega_m P_m^2} + \lambda = 0. \quad (3.22)$$

Therefore,

$$\lambda = \frac{1}{\Omega_m P_m^2}. \quad (3.23)$$

Now, using Equations (3.23) and (3.21), we can write

$$\sum_{m=0}^M \frac{1}{(\Omega_m \lambda)^{1/2}} = P, \quad (3.24)$$

and thus

$$\lambda = \left( \sum_{m=0}^M \frac{1}{P \Omega_m^{1/2}} \right)^2. \quad (3.25)$$

Using  $\lambda$  from Equation (3.25) into (3.23), we can find the power allocation at  $m^{\text{th}}$  relay as given below:

$$P_m = \frac{P}{\Omega_m^{1/2} \sum_{i=0}^M \frac{1}{\Omega_i^{1/2}}}. \quad (3.26)$$

## 4. Path Selection Strategies in Multihop Cooperative Networks

### 4.1. Overview

The amount of energy required to establish a wireless link between two nodes is generally proportional to the distance between the nodes, raised to the path-loss exponent. Due to this relationship, it is beneficial in terms of energy saving, to relay the information through a multihop route. Multihop routing extends the coverage by allowing a node to communicate with nodes that would have otherwise been outside of its transmission range.

What makes the problem of finding the minimum energy cooperative path in wireless networks challenging is the fact that an optimal path could be a combination of cooperative transmissions, multicast, and point-to-point links [Kha07]. The problem of finding the optimal cooperative route from the source node  $s$  to the destination node  $d$ , formulated in [Kha07], can be mapped to a dynamic programming (DP) problem. The state of the system at stage  $k$  is the set  $S_k$ , i.e., the set of nodes that have reliably received the information by the  $k$ th transmission slot. The initial state  $S_0$  is simply  $\{s\}$ , and the termination states are all sets of nodes that contain  $d$ . The decision variable at the  $k$ th stage, determines the set of nodes that will be added to the reliable set in the next transmission slot. The objective is to find a sequence  $\{S_k\}$  that minimizes the total transmission power. [Kha07] refer to the solution to this problem as the *optimal transmission policy*. The optimal transmission policy can be mapped to finding the shortest path in the state space of this dynamica system. The state space can be represented by a graph with all possible states, i.e., all possible subsets of nodes in the network, as its nodes. [Kha07] refers to this graph as the *cooperation graph*.

The nodes in the cooperation graph are connected with arcs representing the possible transitions between the states. As the network nodes are allowed only to either fully cooperate or broadcast, the graph has a special layered structure. Notice that the nodes in the  $k^{\text{th}}$  layer are all the reliable sets of size  $k + 1$ . Hence, in a network with  $n + 1$  nodes, the cooperation graph has  $n + 1$  layers, and the  $k$ th layer has  $\binom{n}{k}$  nodes.

This is illustrated in Figure 9 for a four-node network. The arcs between the nodes in adjacent layers correspond to cooperative links, whereas broadcast links are shown by cross-layer arcs. The nodes are marked as  $s$  and  $d$  for source and destination nodes, respectively, and 1 and 2 for the two relay nodes. Here, the link between the nodes marked  $\{s\}$  and  $\{s, 1\}$  corresponds to a point-to-point link. An example of a multicast (or broadcast) link is the one between the nodes marked  $\{s\}$  and  $\{s, 1, 2\}$ . Finally, the link between  $\{s, 1, 2\}$  and  $\{s, 1, 2, d\}$  demonstrates a case of multipoint-to-point or cooperative scenario. Some of the links are omitted for clarity.

In [Kha07], the signals arrived at the destination are combined and the resulting SNR is required to be greater than a minimum value. In this report, we follow a similar strategy except that, instead of SNR, we focus on the system error rate at the destination since this is a more direct measure of the system performance.

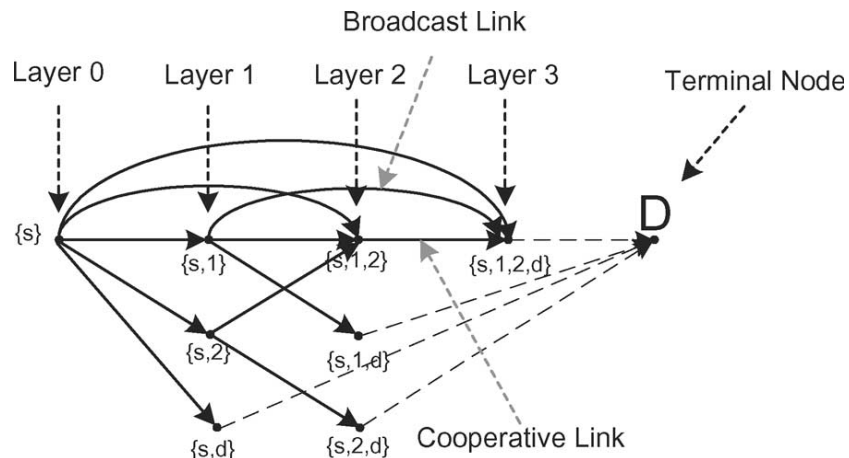


Figure 9: An example of a cooperative graph with 4 nodes [Kha07]

## 4.2. Calculation of Links Costs

In this section, we compute the link costs for successful transmission of information from a set of source nodes  $S = \{s_1, s_2, \dots, s_n\}$  to a set of target nodes  $T = \{t_1, t_2, \dots, t_m\}$ . As in [Kha07], we consider three different cases, namely point-to-point links, point-to-multipoint broadcast links and multipoint-to-point cooperative links. However, unlike [Kha07] in which the minimum received SNR is used for finding best cooperative routing path, we will consider the maximum allowed error rate as the QoS measure. This metric is indeed more meaningful in the design and implementation of wireless networks. Below, we consider each type of communication links separately.

### 4.2.1. Point-to-Point Links

The simplest case in multihop transmission is the case where only one node is transmitting within a time slot to a single target node. In this case,  $S = \{s_1\}$ , and  $T = \{t_1\}$ , i.e.,  $n=m=1$ . With Rayleigh fading, M-PSK or M-QAM modulations and coherent detection, the probability of error is given by [Sim05]

$$P_e^b = \frac{c}{\pi} \int_0^{\pi/2} \frac{\sin^2 \varphi}{\sin^2 \varphi + \frac{g \mu \sigma^2}{2}} d\varphi = \frac{c}{2} \left( 1 - \sqrt{\frac{g \mu \sigma^2}{2 + g \mu \sigma^2}} \right). \quad (4.1)$$

where parameters  $c$  and  $g$  are given by

$$c_{QAM} = 4 \frac{\sqrt{M} - 1}{\sqrt{M} \log_2 M}, \quad c_{PSK} = \frac{2}{\log_2 M},$$

$$g_{QAM} = \frac{3}{M - 1}, \quad c_{PSK} = 2 \sin^2 \left( \frac{\pi}{M} \right).$$

In addition,  $\sigma^2$  represents the variance of the Rayleigh fading channel, and  $\mu$  is the transmitted signal power-to-noise ratio.

We assume that the acceptable bit error rate for each link is less than  $BER_{\max}$ . Using Equation (4.1) and assuming normalized noise power at every receiver, the minimum required power, and hence, the point-to-point link cost  $LC(s_1, t_1)$  is given by

$$LC(s_1, t_1) = P_T = \frac{2}{g \sigma^2} \frac{1}{\frac{1}{\left( 1 - \frac{2 BER_{\max}}{c} \right)^2} - 1}. \quad (4.2)$$

Notice that in the above equation, the point-to-point link cost is proportional to  $1/\sigma^2$  which corresponds to the attenuation of the signal power in the wireless channel between  $s_1$  and  $t_1$ . The link cost, in this case, is therefore, proportional to the square of the distance between  $s_1$  and  $t_1$  under our propagation model.

### 4.2.2. Point-to-Multipoint Broadcast Links

In this case,  $S = \{s_1\}$ , and  $T = \{t_1, t_2, \dots, t_m\}$  (i.e.,  $n = 1$ ,  $m > 1$ ). Thus here,  $m$  simultaneous BER constraints at the receivers must be satisfied.

Assuming that omnidirectional antennas are used, the signal transmitted by the node  $s_1$  is received by all nodes within a radius proportional to the transmission power. Therefore, a broadcast link can be treated as a set of point-to-point links, and the cost of reaching a set of nodes is the maximum of the costs for reaching all the nodes in the target set. The power required for the broadcast transmission, denoted by  $LC(s_1, t_1)$ , is thus given by

$$LC(s_1, T) = \max\{LC(s_1, t_1), LC(s_1, t_2), \dots, LC(s_1, t_m)\}, \quad (4.3)$$

where  $LC(s_1, t_1)$  is given by Equation (4.2).

### 4.2.3. Multipoint-to-Point Cooperative Links

In this case,  $S = \{s_1, s_2, \dots, s_n\}$ , and  $T = \{t_1\}$ , (i.e.,  $n > 1$  and  $m = 1$ ). This case corresponds to a setting in which multiple nodes cooperate to transmit the same information to a single receiver node. Assuming coherent detection at the destination node, signals are combined using maximal ratio combining (MRC). Detection of data symbols is considered possible if the BER at the receiving node is less than a desired value denoted as  $BER_{\max}$ . The total transmission power is

$$P_T = \sum_{i=1}^N P_i.$$

In the following, we calculate the BER at the receiving node  $T$  using a moment generating function (MGF) approach. With Rayleigh fading at each link, the MGF of the  $i$ -th path with the exponential random variable  $\gamma$  is given by

$$M_i(-s) = \int_0^\infty e^{-s\gamma} p_i(\gamma) d\gamma = \frac{1}{1 - s\sigma_i^2}. \quad (4.4)$$

Since the all links are assumed to be independent of each other, the probability of error becomes [Sim05]

$$P_e = \frac{c}{\pi} \int_0^{\pi/2} \prod_{i=1}^n M_i\left(-\frac{g \mu_i}{2 \sin^2 \varphi}\right) d\varphi = \frac{c}{\pi} \int_0^{\pi/2} \prod_{i=1}^n \frac{1}{1 + \frac{g \mu_i}{2 \sin^2 \varphi} \sigma_i^2} d\varphi. \quad (4.5)$$

#### A. Simplified Case

Let us first consider the case where all transmitting nodes have the same distance from the destination, i.e.,  $\sigma_i^2 = \sigma^2$ , for  $i = 1, \dots, n$ . In this case, we can rewrite Equation (4.5) as

$$P_e = \frac{c}{\pi} \int_0^{\pi/2} \frac{1}{\left(1 + \frac{g \mu}{2 \sin^2 \varphi} \sigma^2\right)^n} d\varphi = \frac{c}{\pi} \int_0^{\pi/2} \frac{\sin^{2n} \varphi}{\left(\sin^2 \varphi + \frac{g \mu \sigma^2}{2}\right)^n} d\varphi. \quad (4.6)$$

We will use the following identity to further modify the above equation: [Gra96]

$$I = \int_0^{\pi/2} \frac{\cos^{2n+2} \varphi}{\left(a^2 \cos^2 \varphi + b^2 \sin^2 \varphi\right)^{n+1}} d\varphi = \pi \sum_{k=0}^n \binom{2n-k}{n} \frac{1}{(2a)^{2n-k+1} (a+b)^{k+1}}. \quad (4.7)$$

Using the above identity and some algebraic manipulations, the probability of error can be written as: [Sim05]

$$P_e = \frac{c}{\pi} \int_0^{\pi/2} \frac{\sin^{2n} \varphi}{\left( \sin^2 \varphi + \frac{g \mu \sigma^2}{2} \right)^n} d\varphi = c \left( \frac{1-\lambda}{2} \right)^L \sum_{k=0}^{n-1} \binom{n-1+k}{k} \left( \frac{1+\lambda}{2} \right)^L, \quad (4.8)$$

where

$$\lambda = \sqrt{\frac{g \mu \sigma^2}{2 + g \mu \sigma^2}}. \quad (4.9)$$

In high SNR conditions, in which  $g \mu \sigma^2 \gg 1$ , Equation (4.8) can be approximated as

$$P_e \approx \frac{c}{2^N} \frac{1}{(g \mu \sigma^2)^N} \sum_{k=0}^{N-1} \binom{N-1+k}{k}. \quad (4.10)$$

Note that Equation (4.10) also implies that full spatial diversity of order  $n$  is obtainable.

### B. General Case

Now, we consider the general case of transmitting nodes with different distances from the destination. In this case, no closed-form solution of Equation (4.5) exists. In [Rib04], the following expression for the BER at high SNR's is derived:

$$P_e \rightarrow \frac{\prod_{i=1}^{t+1} (2i-1)}{2(t+1)g^{t+1}t!} \frac{\partial^t p_\gamma(0)}{\partial \gamma^t}, \quad (4.11)$$

where  $\frac{\partial^t p_\gamma(0)}{\partial \gamma^t}$  is the  $t$ -th order derivative of the pdf of the equivalent channel. The derivatives of  $p_\gamma(\gamma)$  up to order  $(t-1)$  are zero. Equation (4.11) is modified using the following proposition [Rib05] :

**Proposition 4.1:** Consider a finite set of nonnegative random variables  $\{\gamma_1, \gamma_2, \dots, \gamma_n\}$  whose pdf's  $p_1, p_2, \dots, p_n$  have nonzero values at zero, and denote these values as  $p_1(0), p_2(0), \dots, p_n(0)$ . If the random variable  $\gamma$  is the sum of the components of the set  $\{\gamma_1, \gamma_2, \dots, \gamma_n\}$ ,  $\gamma = \sum_{i=1}^n \gamma_i$ , then all the derivatives of  $p_\gamma$  evaluated at zero up to order  $(n-2)$  are zero, while the  $(n-1)$ th order derivative is given by

$$\frac{\partial^{n-1} p_\gamma}{\partial \gamma^{n-1}}(0) = \prod_{i=1}^n p_i(0). \quad (4.12)$$

The poof of the above proposition is addressed in [Rib05].

Using the above proposition in Equation (4.11), we can write:

$$P_e \rightarrow c \frac{\prod_{i=1}^n (2i-1)}{2^n g^n (n-1)!} \prod_{i=1}^n p_i(0). \quad (4.13)$$

An exponential distribution with mean  $\mu_i \sigma_i^2$  has a value of  $1/\sigma_i^2$  at zero. This will yield the following approximation:

$$P_e \approx c \frac{\prod_{i=1}^n (2i-1)}{2^n g^n n!} \prod_{i=1}^n \frac{1}{\mu_i \sigma_i^2}. \quad (4.14)$$

Note that if  $\sigma_i^2 = \sigma^2$ , for  $i = 1, \dots, n$ , the expression derived in Equation(4.14) is equivalent to Equation (4.8).

To obtain the link cost for the multipoint-to-point case, we need to consider the following constrained optimization problem:

$$\min \sum_{i=1}^n P_i \quad (4.15)$$

$$\text{s.t. } c \frac{\prod_{i=1}^n (2i-1)}{2g^n n!} \prod_{i=1}^n \frac{1}{P_i \sigma_i^2} \leq \text{BER} \quad (4.16)$$

Similar to the previous cases, it can be proved that the solution to the above optimum power allocation problem, is unique.

**Proposition 4.2:** For the set of  $n$  selected relays in the network, the optimum transmission powers satisfy the following equations:

$$P_i = \frac{\Psi(n)}{\text{BER}} \frac{1}{\sigma_i^2} \prod_{\substack{k=1 \\ k \neq i}}^n \frac{1}{P_k \sigma_k^2} \quad (4.17)$$

where

$$\Psi(n) = c \frac{\prod_{i=1}^n (2i-1)}{2g^n n!} . \quad \text{for } i = 1, \dots, n . \quad (4.18)$$

**Proof:** The Lagrangian in Equations (4.17) and (4.18) is given by

$$L(P_1, \dots, P_n) = \sum_{i=1}^n P_i + \lambda f(P_1, \dots, P_n), \quad (4.19)$$

where

$$f(P_1, \dots, P_n) = \Psi(n) \prod_{i=1}^n \frac{1}{P_i \sigma_i^2} - \text{BER} . \quad (4.20)$$

For nodes  $i = 1, \dots, n$  with nonzero transmitter powers, the Kuhn-Tucker conditions are

$$\frac{\partial}{\partial P_i} L(P_1, \dots, P_n) = 1 + \lambda \frac{\partial}{\partial P_i} f(P_1, \dots, P_n) = 0, \quad (4.21)$$

where

$$\frac{\partial}{\partial P_i} f(P_1, \dots, P_n) = -\Psi(n) \frac{1}{P_i^2 \sigma_i^2} \prod_{\substack{k=1 \\ k \neq i}}^n \frac{1}{P_k \sigma_k^2} \quad (4.22)$$

Using Equations (4.21) and (4.22), we get

$$P_i^2 = \lambda \Psi(n) \frac{1}{\sigma_i^2} \prod_{\substack{k=1 \\ k \neq i}}^n \frac{1}{P_k \sigma_k^2}, \quad \text{for } i = 1, \dots, n . \quad (4.23)$$

Since strong duality condition [Boy04] holds for convex optimization problems, we can write  $\lambda^* f(P_1^*, \dots, P_n^*) = 0$ . If we assume Lagrange multiplier has a positive value, we have  $f(P_1^*, \dots, P_n^*) = 0$ , which is equivalent to

$$\text{BER} = \Psi(n) \prod_{i=1}^n \frac{1}{P_i \sigma_i^2} \quad (4.24)$$

Using Equations (4.23) and (4.24), we can find the Lagrange multiplier as

$$\lambda = \frac{P_i}{\text{BER}}. \quad (4.25)$$

Substituting  $\lambda$  from Equation (4.25) into (4.23) yields the desired results, i.e.,

$$P_i = \frac{\Psi(n)}{\text{BER}} \frac{1}{\sigma_i^2} \prod_{\substack{k=1 \\ k \neq i}}^n \frac{1}{P_k \sigma_k^2}. \quad (4.26)$$

■

**Theorem 4.1:** The values for the optimal allocated powers,  $P_1^*, \dots, P_n^*$ , in the constrained optimization problem stated in Equations (4.15) and (4.16) are identical and are given by

$$P_i^* = \left( \frac{\Psi(n)}{\text{BER}} \prod_{k=1}^n \frac{1}{\sigma_k^2} \right)^{\frac{1}{n}}. \quad (4.27)$$

**Proof:** The constrained optimization problem of Equations (4.15) and (4.16) has a unique solution. It is readily verified that  $P_1^* = P_1^* = \dots = P_n^*$  is a solution for the set of the Equations in (4.17), which in turn leads to Equation (4.27).

The resulting cooperative link cost  $LC(S, t_1)$ , defined as the total transmission power allocated optimally, is, therefore, given by

$$LC(S, t_1) = \sum_{i=1}^n P_i^* = n \left( \frac{\Psi(n)}{\text{BER}} \prod_{k=1}^n \frac{1}{\sigma_k^2} \right)^{\frac{1}{n}}. \quad (4.28)$$

### 4.3. Optimal Cooperative Route Selection

The cost for different links calculated in the previous section can now be used for the selection of the cooperative route from the source to the destination as outlined in Section 4.1. As stated earlier, the problem of finding the optimal can be mapped to a dynamic programming (DP) problem [Kha07].

Recall that in a cooperative graph, the state of the system at stage  $k$  is the set  $S_k$  defined to be the set of nodes that have reliably received the information by the  $k^{\text{th}}$  transmission slot. The set of nodes that will be added to the reliable set in the next transmission slot, denoted by  $U_k$ , has the following relationship with  $S_{k+1}$ :

$$S_{k+1} = S_k \cup U_k, \quad k = 1, 2, \dots. \quad (4.29)$$

The objective is to find a sequence  $\{U_k\}$  or  $\{S_k\}$  that minimizes the total transmitted power  $P_T$  given by

$$P_T = \sum_k LC(S_k, U_k) = \sum_k LC(S_k, S_{k+1} - S_k). \quad (4.30)$$

The result will be what is referred to as the *optimal transmission policy* [Kha07].

### 4.4. Analytical and Numerical Results

We will use the following metric for measuring the energy saving offered by the cooperative route selection technique outlined in the previous section over a non-cooperative scheme:

$$\text{Energy Savings} = \frac{P_T(\text{noncoop}) - P_T(\text{coop})}{P_T(\text{noncoop})}, \quad (4.31)$$

where  $P_T$  denotes the total transmission power in each case.

In this section, we present analytical and numerical results for achievable energy savings in a line network. In particular, we consider a *regular line* topology where the nodes are located at equal (unity) distances from each other on a straight line as shown in Figure 10.

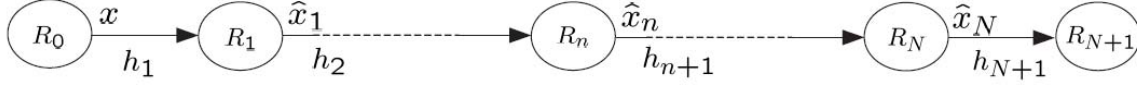


Figure 10: A line network

In this case, the optimal non-cooperative routing strategy is to always send the information to the next nearest node in the direction of the destination.  $N$  relay stations are denoted by  $R_1, R_2, \dots, R_N$ . Each  $R_n$ ,  $n = 1, 2, \dots, N$ , transmits its detected symbol  $\hat{x}_n$ , while  $R_0$  transmits the user data symbol  $\hat{x}_0 = x$ . The received symbol at each node  $R_n$ ,  $n = 1, 2, \dots, N+1$  is now renamed as  $y_n = h_n \hat{x}_{n-1} + z_n$ , where  $h_n$  denotes the fading coefficient of the link between  $R_n$  and  $R_{n-1}$ . We assume that relay stations transmit their signals over mutually orthogonal channels.

The BER from  $R_{n-1}$  to  $R_n$  is denoted as  $P_{n-1,n}^b$ . The BER  $P_n^b$  at node  $R_n$  is affected by all previous  $n-1$  hops and can be iteratively calculated using the following recursive formula:  $P_n^b = (1 - P_{n-1,n}^b)P_{n-1,n}^b + (1 - P_{n-1,n}^b)P_{n-1}^b$ , with  $P_0^b = 0$ . The end-to-end BER at the destination can then be written as  $P_{N+1}^b = (1 - P_N^b)P_{N,N+1}^b + (1 - P_{N,N+1}^b)P_N^b$ .

Given the assumption regarding equally spaced nodes, we require identical error rate performance,  $BER_{\max}$ , at all links. With BPSK modulation for which  $g = 2$  and  $c = 1$ , the required power at each link becomes:

$$P_{\text{Link}}(\text{noncoop}) = \frac{2}{g \sigma^2} \frac{1}{\left(1 - \frac{2 BER_{\max}}{c}\right)^{-2} - 1} \approx \frac{c}{2 g BER_{\max}} \frac{1}{\sigma^2}. \quad (4.32)$$

and thus the total power for the non-cooperative scheme is given by

$$P_T(\text{noncoop}) \approx N \frac{c}{2 g BER_{\max}} \frac{1}{\sigma^2} \quad (4.33)$$

Let us now consider the total power due to the cooperative signal transmission. At each transmission slot, all nodes that have received the information cooperate to send the information to the next node along the same route as the non-cooperative path. After the  $m$ th transmission slot, the reliable set is  $S_m = \{s, 1, \dots, m\}$ , and these nodes cooperate to send the information to the next node,  $(m+1)$ . The corresponding link cost (from Equation (4.28)) is given by

$$LC(S_m, m+1) = \sum_{i=1}^{m+1} P_i = (m+1) \left( \frac{\Psi(m+1)}{BER} \prod_{k=1}^{m+1} k^2 \right)^{\frac{1}{m+1}}. \quad (4.34)$$

where  $\text{BER}_{\max}$  is the required BER at each stage for a reliable data transmission, and  $\Psi(n)$  was defined in Equation (4.18).

Using  $\Psi(m+1)$  from Equation (4.18) in the above expression, we can write:

$$LC(S_m, m+1) = (m+1) \left( c \frac{\prod_{i=1}^{m+1} (2i-1)}{2 g^{m+1} \text{BER}} (m+1)! \right)^{\frac{1}{m+1}} \quad (4.35)$$

The total transmission power for the cooperative strategy is then given by

$$P_T(\text{coop}) = \sum_{m=0}^{N-1} LC(S_m, m+1) . \quad (4.36)$$

We can now use Equations (4.33) and (4.35) to obtain the Energy Saving offered by the cooperative scheme over the non-cooperative approach, as shown below:

$$\text{Energy Savings} = \frac{P_T(\text{noncoop}) - P_T(\text{coop})}{P_T(\text{noncoop})} = \frac{N \frac{c}{2 g \text{BER}} - \sum_{m=0}^{N-1} LC(S_m, m+1)}{N \frac{c}{2 g \text{BER}}} . \quad (4.37)$$

Simplifying Equation (4.37), we get

$$\text{Energy Savings} = 1 - \frac{2 \text{BER}}{c N} \sum_{m=0}^{N-1} (m+1) \left( c \frac{\prod_{i=1}^{m+1} (2i-1)}{2 \text{BER}} (m+1)! \right)^{\frac{1}{m+1}} . \quad (4.38)$$

#### 4.4.1. U-Cooperation

In calculating the Energy Saving in Equation (4.38), we had assumed that all previous nodes cooperate in the (m+1)th step of transmission. We can reduce the total transmission power by considering the cooperation at a given stage to be from the last U stages. We will refer to this method as the U-cooperation scheme. The Energy Saving for this modified cooperative scheme is given by

$$\text{Energy Savings} = 1 - \frac{2 \text{BER}}{c N} \left[ \sum_{m=0}^{U-1} (m+1) \left( c \frac{\prod_{i=1}^{m+1} (2i-1)}{2 \text{BER}} (m+1)! \right)^{\frac{1}{m+1}} + (N-U) U \left( c \frac{\prod_{i=1}^U (2i-1)}{2 \text{BER}} U! \right)^{\frac{1}{U}} \right] \quad (4.39)$$

In Figure 11, the Energy Saving of the cooperative network over the non-cooperative signal transmission is plotted for the number of nodes ranging from 2 to 40, and for the BER values of  $10^{-3}$  and  $10^{-4}$ . The plot shows that the significant saving in energy is achieved in the cooperative multi-hop scheme, especially for the lower error rate case. The plots also reveal that maximum saving is obtained when the number of nodes is about 4 or 5. This indicates that the U-cooperative approach with U in the range of 4-5 could provide substantial additional saving. In Figure 12, the energy saving due to a

U-cooperative scheme is plotted, where  $U = 4$  and  $U = 5$  corresponding to  $\text{BER}_{\max}$  equal to  $10^{-3}$  and  $10^{-4}$ , respectively. Notice that, in both cases, the energy saving is about 90% and approaches unity as the number of nodes increases.

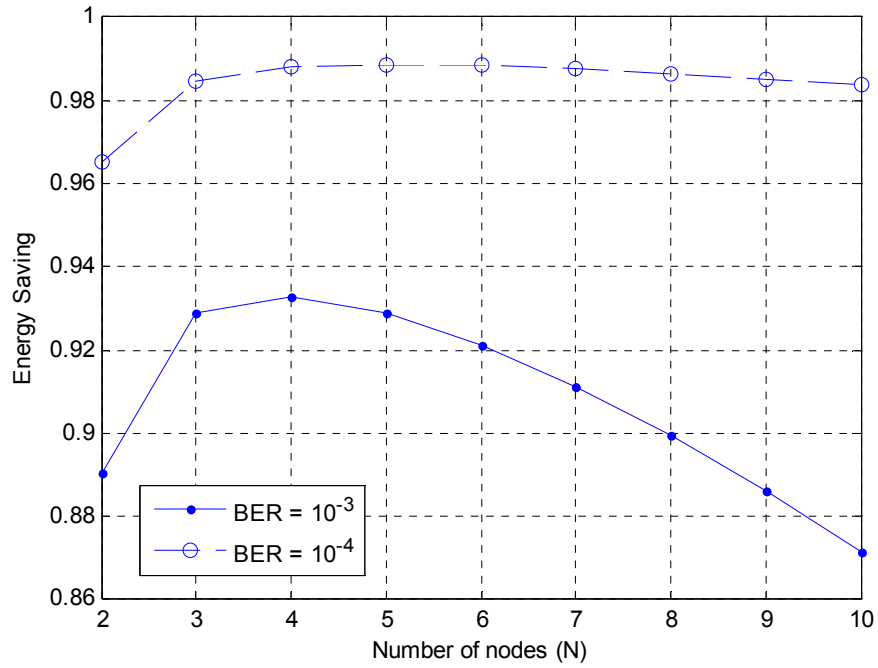


Figure 11: Energy saving vs. the number of nodes

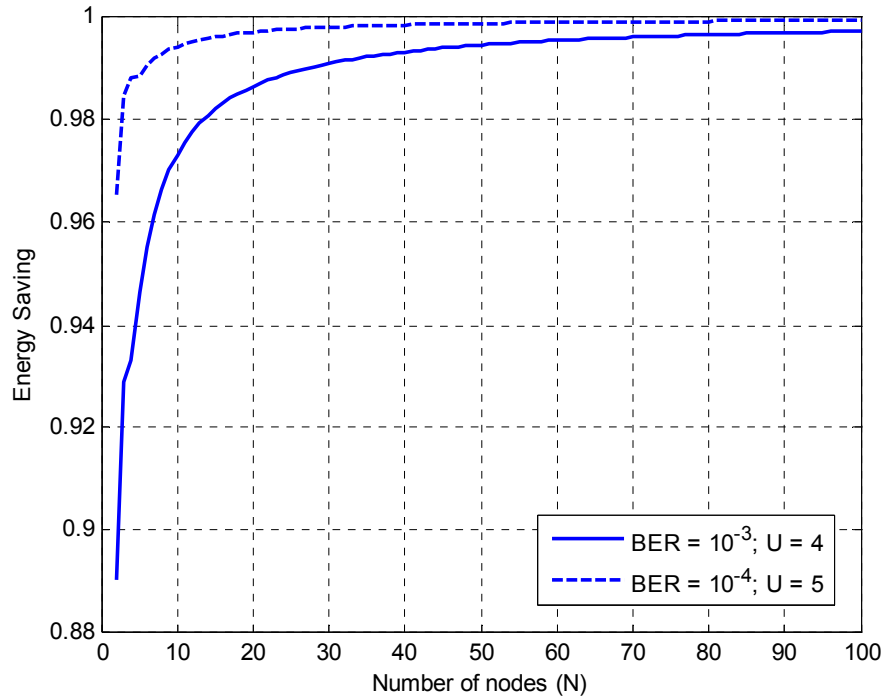


Figure 12: Energy saving vs. the number of nodes for the U-cooperative scheme

## 5. Conclusions

In this report, we proposed techniques for setting the optimal transmission power for relay nodes in single-hop as well as multi-hop wireless cooperative / sensor networks. In each case, this was done by minimizing the total power subject to system SER remaining below an acceptable value. Alternatively, we explored cases in which the SER was minimized subject to the total transmission power being constant. We developed configuration algorithms for single-hop multi-networks which incorporate the optimal power allocation methodology.

For single-hop networks, we studied a network lifetime maximization scheme, such that, when the battery power of a relay is depleted, the node is removed from the set of the cooperative terminals. The signal is then routed by utilizing other nodes as relays in a new cooperative formation.

We also investigated the problem of selecting the optimal cooperative route in multi-hop networks. Here, we calculated the link cost for three types of communication links, namely point-to-point, broadcast and multi-point to point links. Using these link costs, the optimal route, which has the lowest cost or transmission power, can be found using a dynamic programming approach. We also studied an alternate routing strategy such that, at a given stage, cooperation was considered to only arise from the last U stages. This so-called U-cooperation strategy resulted in significant increases in energy saving in the cooperative case.

## References

- [Alz03] Khaled Alzoubi, Xiang-Yang Li, Yu Wang, Peng-Jun Wan, and Ophir Frieder, "Geometric spanners for wireless ad hoc networks," *IEEE Transactions On Parallel And Distributed System*, vol. 14, May 2003.
- [Ang03] P. Anghel, G. Leus, and M. Kaveh, "Multi-user space-time coding in cooperative networks," in *Proc. IEEE Int. Conf. Acoustics, Speech, Signal Processing*, vol. 4, 2003, pp. 73–76.
- [Boy04] J. Boyer, D. D. Falconer, and H. Yanikomeroglu, "Multihop diversity in wireless relaying channels," *IEEE Trans. on Commun.*, vol. 52, no. 10, pp. 1820–1830, Oct. 2004.
- [Bro98] J. Broch et al., "A performance comparison of multihop wireless ad hoc network routing protocols," in *Proc. IEEE/ACM MOBICOM*, 1998, pp. 85–97.
- [Cha00] Jae-Hwan Chang and Leandros Tassiulas, "Energy conserving routing in wireless ad hoc networks," in *INFOCOM*, 2000.
- [Cer02] Alberto Cerpa and Deborah Estrin, "Ascent: Adaptive self-configuring sensor networks topologies," in *INFOCOM*, 2002.
- [Che01] Benjie Chen, Kyle Jamieson, Hari Balakrishnan, and Robert Morris, "Span: An energy-efficient coordination algorithm for topology maintenance in ad hoc wireless networks," in *MobiCom*, 2001.
- [Che05] Y. Chen and Q. Zhao, "Maximizing the lifetime of sensor network using local information on channel state and residual energy," in *Proceedings of the Conference on Information Science and Systems (CISS)*, 2005.
- [Cov91] T. M. Cover and J. Thomas, *Elements of Information Theory*, John Wiley & Sons, Inc., 1991.
- [Dos02a] Sheetakumar Doshi, Shweta Bhandare, and Timothy X Brown, "An on-demand minimum energy routing protocol for a wireless ad hoc network," *SIGMOBILE Mob. Comput. Commun. Rev.*, vol. 6, no. 3, pp. 50–66, 2002.
- [Dos02b] Sheetakumar Doshi and Timothy X Brown, "Minimum energy routing schemes for a wireless ad hoc network," in *INFOCOM*, 2002.
- [Ema02] V. Emamian, P. Anghel, and M. Kaveh, "Multi-user spatial diversity in a shadow-fading environment," in *Proc. IEEE Vehicular Technology Conf.*, vol. 1, 2002, pp. 573–576.
- [Han02] M. O. Hasna and M.-S. Alouini, "Performance analysis of two-hop relayed transmissions over Rayleigh-fading channels," in *Proc. Vehicular Technology Conf.*, Birmingham, AL, 2002, pp. 1992–1996.
- [Hon07] Y.-W. Hong, W.-J. Huang, F.-H. Chiu, and C.-C. J. Kuo, "Cooperative communications in resource-constrained wireless networks," to appear in *IEEE Signal Processing Magazine*, May 2007.
- [Hua07] W. Huang, Y. Hong, and C. J. Kuo, "Lifetime maximization for amplify-and-forward cooperative networks," in *Proceedings of the IEEE Wireless Communications and Networking Conference (WCNC)*, 2007.
- [Joh96] D. Johnson and D. Maltz, "Dynamic source routing in ad hoc wireless networks," in *Mobile Computing*, T. Imielinski and H. Korth, Eds. Norwell, MA: Kluwer, 1996, ch. 5, pp. 153–181.

- [Joh99] P. Johansson et al., "Scenario-based performance analysis of routing protocols for mobile ad hoc networks," in Proc. IEEE/ACM MOBICOM, 1999, pp. 195–206.
- [Kah77] R. Kahn, "The organization of computer resources into a packet radio network," IEEE Trans. Commun., vol. COM-25, pp. 169–178, Jan. 1977.
- [Kha07] Khandani A., J. Abounadi, E. Modiano, L. Zheng, "Cooperative Routing in Static Wireless Networks," in IEEE Trans. Commun.
- [Lan04] J. N. Laneman, D. N. C. Tse and G. W. Wornell., "Cooperative diversity in wireless networks: efficient protocols and outage behavior", IEEE Trans. Information Theory, vol. 50, no. 12, December 2004.
- [Li01] Qun Li, Javed Aslam, and Daniela Rus, "Online power-aware routing in wireless ad-hoc networks," in MobiCom, 2001.
- [Li06] F. Li, K. Wu, A. Lippman, "Energy-Efficient Cooperative Routing in Multi-hop Wireless Ad Hoc Networks," in IEEE International Performance, Computing, and Communications Conference (IPCCC), April 2006.
- [Mos05] T. Moscibroda and R. Wattenhofer, "Maximizing the lifetime of dominating sets," in WMAN, 2005.
- [Nin03] Ning Li, Jennifer C. Hou, and Lui Sha, "Design and analysis of an mst-based topology control algorithm," in INFOCOM, 2003.
- [PCM01] IEEE Pers. Commun. Mag.: Special Issue on Advances in Mobile Ad Hoc Networking, vol. 8, Jan. 2001.
- [Ram00] Ram Ramanathan and Regina Hain, "Topology control of multihop wireless networks using transmit power adjustment," in INFOCOM, 2000.
- [Rib04] A. Ribeiro, A. Cai, G. B. Giannakis, "Symbol Error Probability for General Cooperative Links", ICC 2004, Paris, march 2004.
- [Rib05] A. Ribeiro, A. Cai, G. B. Giannakis, "Symbol Error Probability for General Cooperative Links", *IEEE Trans. on Wireless Communications*, vol. 4, no. 3, May 2005.
- [San04] Arvind Sankar and Zhen Liu, "Maximum lifetime routing in wireless ad-hoc networks," in INFOCOM, 2004.
- [Sim05] M. K. Simon and M.-S. Alouini, Digital Communication over Fading Channels: A Unified Approach to Performance Analysis. New York, NY: Wiley, 2005.
- [Sin98] Suresh Singh, Mike Woo, and C. S. Raghavendra, "Power-aware routing in mobile ad hoc networks," in Proceedings of the 4th annual ACM/IEEE international conference on Mobile computing and networking, 1998.
- [Sta03] John A. Stankovic, Tarek Abdelzaher, Chenyang Lu, Lui Sha, and Jennifer Hou, "Real-time communication and coordination in embedded sensor networks," Proceedings of the IEEE, vol. 91, no. 7, 2003.
- [Swe02] Swetha Narayanaswamy, Vikas Kawadia, R. S. Sreenivas, and P. R. Kumar, "Power control in ad-hoc networks: Theory, architecture, algorithm and implementation of the compow protocol," in European Wireless Conference, 2002.
- [Su05] W. Su, A. K. Sadek, K.J.R. Liu, "SER performance analysis and optimum power allocation for decode-and-forward cooperation protocol in wireless networks", 2005 *IEEE Wireless Communications and Networking Conference (WCNC)*, New Orleans, March 2005.
- [Vaz05] J. Vazifehdan, H. Shafiee, "Optimal Configuration of Wireless Cooperative Networks," in Conference on Wireless and Optical Communications Networks, Dubai, UAE, March 6-8, 2005.
- [Vaz07] J. Vazifehdan, H. Shafiee, "Multi-Branch Configuration of Dynamic Wireless Cooperative Networks, ICSPC 2007, Dubai, UAE
- [Vik03] Vikas Kawadia and P. R. Kumar, "Power control and clustering in ad hoc networks," in INFOCOM, 2003.
- [Vol99] Volkan Rodoplu and Teresa H. Meng, "Minimum energy mobile wireless networks," IEEE J. Selected Areas in Communications, vol. 17(8), 1999.
- [Wan07] T. Wang, A. Cano, G. B. Giannakis, and N. Laneman, "High-Performance Cooperative Demodulation with Decode-and-Forward Relays," IEEE Transactions on Communications, vol. 55, no. 7, pp., July 2007.
- [Wie00] Wieselthier J., G. Nguyen, A. Ephremides, "On the Construction of Energy-Efficient Broadcast and Multicast Trees in Wireless Networks," IEEE INFOCOM 2000.
- [Xu00] Ya Xu, John Heidemann, and Deborah Estrin, "Adaptive energy-conserving routing for multihop ad hoc networks," Research Report 527, USC, October 2000.
- [Xu01] Ya Xu, John Heidemann, and Deborah Estrin, "Geography-informed energy conservation for ad hoc routing," in MobiCom, 2001.
- [Xia03] Xiaorui Wang, Guoliang Xing, Yuanfang Zhang, Chenyang Lu, Robert Pless, and Christopher D. Gill, "Integrated coverage and connectivity configuration in wireless sensor networks," in Sensys, 2003.